TATA INSTITUTE OF FUNDAMENTAL RESEARCH

GS 2019 Admission Test in Physics

DECEMBER 9, 2018

Instructions for candidates for Ph.D. programme in Physics

PLEASE READ THESE INSTRUCTIONS CAREFULLY BEFORE YOU ATTEMPT THE QUESTIONS

- You may NOT keep with you any books, papers, mobile phones or any electronic devices which can be used to get/store information. **Use of scientific, non-programmable calculators is permitted**. Calculators which plot graphs are NOT allowed. Multiple-use devices, such as smart phones, etc. CANNOT be used as calculators.

- This test consists of TWO sections.
    - **SECTION A comprises 25 questions, numbered Q. 1 — Q. 25**.

      These are questions on basic topics.
    - **SECTION B comprises 15 questions, numbered Q. 1 — Q. 15**.

      These may require somewhat more thought/knowledge.

- ALL questions are Multiple-Choice Type. In each case, ONLY ONE option is correct. Answer them by clicking the radio button next to the relevant option.

- If your calculated answer does not match any of the given options exactly, you may mark the closest one if it is reasonably close.

- The **grading scheme** will be as follows:

  Section A : **+3** marks if correct; **–1** mark if incorrect; **0** marks if not attempted

  Section B : **+5** marks if correct; **0** marks if incorrect or not attempted,
  i.e. NO negative marks.

- The invigilators will supply you with paper sheets for rough work.

- **Do NOT ask the invigilators for clarifications regarding the questions**. They have been instructed not to respond to any such queries. In case a correction/clarification is deemed necessary, it will be announced in the examination hall.

- You can get a list of **useful physical constants** on the reverse of this page. Make sure to use only these values in answering the questions, especially where the options are numerical.

LIST OF USEFUL CONSTANTS

Physical Constants

Symbol	Name/Definition	Value	(SI Units)	Value	(Other Units)
c	speed of light in vacuum	3×10^8	m s^{-1}		
$\hbar$	reduced Planck constant ($= h/2\pi$)	1.04×10^{-34}	J s		
G_N	gravitational constant	6.67×10^{-11}	m^3 kg^{-1} s^{-2}		
$M_\odot$	solar mass	1.989×10^{30}	kg		
ε_0	permittivity of free space	8.85×10^{-12}	F m^{-1}		
μ_0	permeability of free space	$4\pi \times 10^{-7}$	N A^{-2}		
e	electron charge (magnitude)	1.6×10^{-19}	C		
m_e	electron mass	9.1×10^{-31}	kg	$= 0.511$	MeV/c^2
a_0	Bohr radius	0.51	Å		
	ionisation potential of H atom	13.6	eV		
N_A	Avogadro number	6.023×10^{23}	mol^{-1}		
k_B	Boltzmann constant	1.38×10^{-23}	J K^{-1}	$= 8.6173 \times 10^{-5}$	eV K^{-1}
$R = N_A k_B$	gas constant	8.31	J mol^{-1} K^{-1}		
$\gamma = C_p/C_V$	ratio of specific heats: monatomic gas	1.67			
	diatomic gas	1.40			
σ	Stefan-Boltzmann constant	5.67×10^{-8}	W m^{-2} K^{-4}		
α	fine structure constant ($= e^2/4\pi\varepsilon_0\hbar c$)	1/137			
g	acceleration due to gravity	9.8	m s^{-2}		
R_E	radius of the Earth	6.4×10^3	Km		
R_S	radius of the Sun	7×10^5	Km		
m_p	proton mass ($\approx 2000\, m_e$)	1.7×10^{-27}	kg	$= 938.2$	MeV/c^2
m_n	neutron mass ($\approx 2000\, m_e$)	1.7×10^{-27}	kg	$= 939.6$	MeV/c^2

Conversion Factors

Symbol	Name/Definition	Value	(SI Units)	Value	(Other Units)
$\hbar c$	conversion constant			197.3	MeV fm
1 A.U.	mean distance of Earth from Sun	1.5×10^9	km		
1 a.m.u.	atomic mass unit	1.6×10^{-27}	kg	$= 931.5$	MeV/c^2
1 eV	electron Volt	1.6×10^{-19}	J		
1 T	Tesla	10^4	gauss		
1 bar	mean atmospheric pressure at 0^0 C	1.01×10^5	Pa (= N m^{-2})		
1 Å	Ångstrom unit	10^{-8}	cm		
1 kWh	Commercial energy unit	3.6×10^6	J		

Section A

This Section consists of 25 questions. All are of multiple-choice type. Mark only one option on the ORS provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +3 marks, an incorrect answer will get –1 mark.

1. Consider the surface defined by $ax^2 + by^2 + cz + d = 0$, where a, b, c and d are constants. If $\hat{n}_1$ and $\hat{n}_2$ are unit normal vectors to the surface at the points $(x, y, z) = (1,1,0)$ and $(0,0,1)$ respectively, and $\hat{m}$ is a unit vector normal to both $\hat{n}_1$ and $\hat{n}_2$, then $\hat{m} =$

(a) $\dfrac{-a\,\hat{\boldsymbol{\imath}} + b\,\hat{\boldsymbol{\jmath}}}{\sqrt{a^2 + b^2}}$

(b) $\dfrac{b\,\hat{\boldsymbol{\imath}} - a\,\hat{\boldsymbol{\jmath}}}{\sqrt{a^2 + b^2}}$

(c) $\dfrac{2a\,\hat{\boldsymbol{\imath}} + 2b\,\hat{\boldsymbol{\jmath}} - c\,\hat{\boldsymbol{k}}}{\sqrt{4a^2 + 4b^2 + c^2}}$

(d) $\dfrac{a\,\hat{\boldsymbol{\imath}} - b\,\hat{\boldsymbol{\jmath}} + c\,\hat{\boldsymbol{k}}}{\sqrt{a^2 + b^2 + c^2}}$

2. The eigenvalues of a 3×3 matrix $\mathbb{M}$ are

$$\lambda_1 = 2 \qquad \lambda_2 = -1 \qquad \lambda_3 = 1$$

and the eigenvectors are

$$e_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad e_2 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \qquad e_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

The matrix $\mathbb{M}$ is

(a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

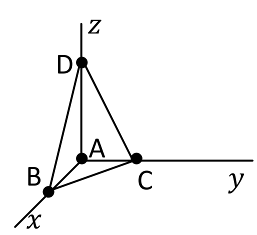
(b) $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

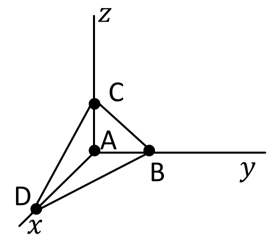
3. Which of the following operations will transform a tetrahedron ABCD with vertices as listed below

	x	y	z
A	0	0	0
B	1	0	0
C	0	1	0
D	0	0	2



into a tetrahedron ABCD with vertices as listed below

	x	y	z
A	0	0	0
B	0	1	0
C	0	0	1
D	2	0	0



up to suitable translation ?

(a) A rotation about x axis by $\pi/2$, then a rotation about z axis by $\pi/2$

(b) A reflection in the xy plane, then a rotation about x axis by $\pi/2$

(c) A reflection in the yz plane, then a reflection in the xy plane

(d) A rotation about y axis by $\pi/2$, then a reflection in the xz plane

4. A British coin has a portrait of Queen Elizabeth II on the 'heads' side and 'ONE POUND' written on the 'tails' side, while an Indian coin has a portrait of Mahatma Gandhi on the 'heads' side and '10 RUPEES' written on the 'tails' side (see below).

These two coins are tossed simultaneously twice in succession.

The result of the first toss was 'heads' for both the coins. The probability that the result of the second toss had a '10 RUPEES' side is

(a) 1/2 (b) 4/7 (c) 3/5 (d) 2/3

5. A set of polynomials of order n are given by the formula

$$P_n(x) = (-1)^n \exp\left(\frac{x^2}{2}\right) \frac{d^n}{dx^n} \exp\left(-\frac{x^2}{2}\right)$$

The polynomial $P_7(x)$ of order $n = 7$ is

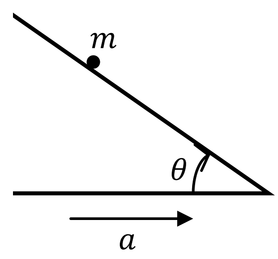
(a) $x^7 - 21x^5 + 105x^4 + 35x^3 - 105x$

(c) $x^7 - 21x^5 + 105x^3 - 105x + 21$

(b) $x^6 - 21x^5 + 105x^4 - 105x^3 + 21x^2 + x$

(d) $x^7 - 21x^5 + 105x^3 - 105x$

6. A particle of mass m is placed on an inclined plane making an adjustable angle θ with the horizontal, as shown in the figure. The coefficient of friction between the particle and the inclined plane is μ.



If the inclined plane is moving horizontally with a uniform acceleration $a < g/\mu$ (see figure), the value of θ for which the particle will remain at rest on the plane is

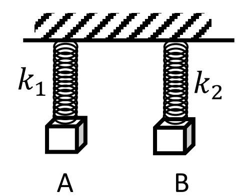
(a) $\theta = \tan^{-1}\left(\frac{\mu g + a}{g + \mu a}\right)$

(c) $\theta = \tan^{-1}\left(\frac{\mu g + a}{g - \mu a}\right)$ ✓

(b) $\theta = -\cot^{-1}\left(\frac{\mu a + g}{a + \mu g}\right)$

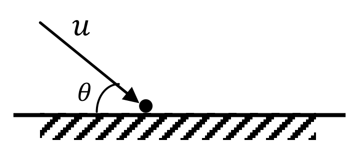
(d) $\theta = \cot^{-1}\left(\frac{\mu a - g}{a + \mu g}\right)$

7. Two bodies A and B of equal mass are suspended from two rigid supports by separate massless springs having spring constants k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of oscillations of A to that of B is



(a) k_1/k_2 (b) k_2/k_1 (c) $\sqrt{k_1/k_2}$ (d) $\sqrt{k_2/k_1}$ ✓

8. A particle of mass m is bounced on the ground with a velocity u making an angle of θ with the ground. The coefficient of restitution for collisions between the particle and the ground is ε and frictional effects are negligible both on the ground and in the air. The horizontal distance travelled by the particle from the point of initial impact till it begins to slide along the ground is



(a) $\frac{u^2}{2g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$

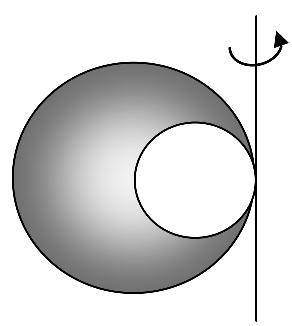
(c) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\tan 2\theta$

(b) $\frac{u^2}{g}\left(\frac{1}{1-\varepsilon}\right)\sin\theta$

(d) $\frac{u^2}{g}\left(\frac{\varepsilon}{1-\varepsilon}\right)\sin 2\theta$ ✓

9. The three-dimensional object sketched on the right is made by taking a solid sphere of uniform density (shaded) with radius R, and scooping out a spherical cavity (unshaded) as shown, which has diameter R.

If this object has mass M, its moment of inertia about the tangential axis passing through the point where the spheres touch (as shown in the figure) is



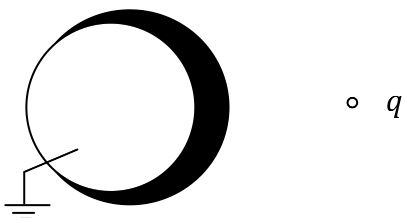
(a) $\frac{3}{16}MR^2$ (b) $\frac{62}{35}MR^2$ (c) $\frac{31}{70}MR^2$ (d) $\frac{31}{20}MR^2$ ✓

10. Consider three straight, coplanar, parallel wires of infinite length where the distance between adjacent wires is d. Each wire carries a current I in the same direction. The perpendicular distance from the middle wire (on either side) where the magnetic field vanishes is

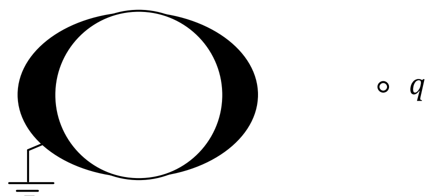
(a) $d/\sqrt{3}$ (b) $2d/3$ (c) $d/3$ (d) $2d/\sqrt{3}$

11. A point charge $q < 0$ is brought in front of a grounded conducting sphere. If the induced charge density on the sphere is plotted such that that the thickness of the black shading is proportional to the charge density, the correct plot will most closely resemble

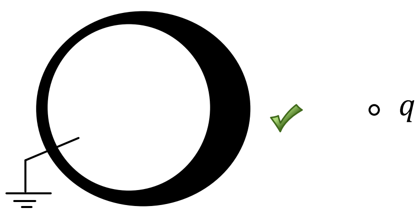
(a)



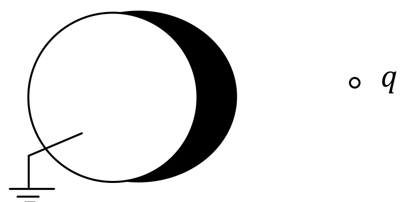
(c)



(b)

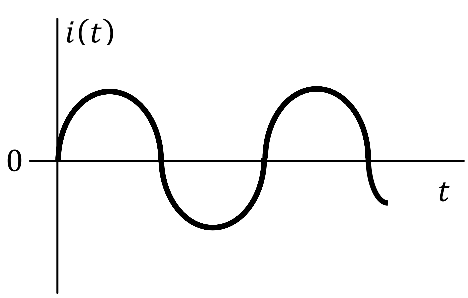


(d)

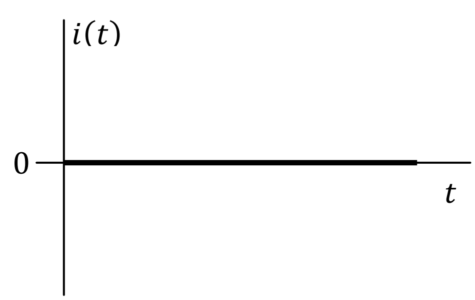


12. A circular coil of conducting wire, of radius a and n turns, is placed in a uniform magnetic field $\vec{B}$ along the axis of the coil and is then made to undergo simple harmonic oscillations along the direction of the axis. The current through the coil will be best described by

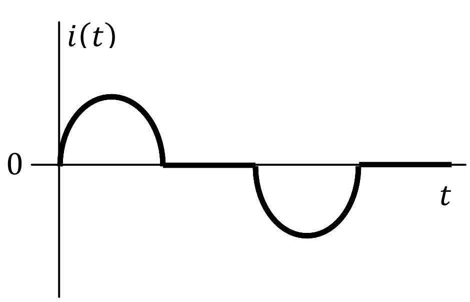
(a)



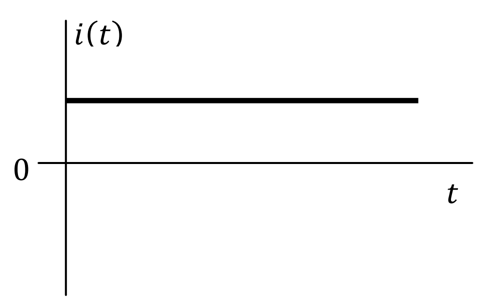
(c)



(b)



(d)



13. A plane electromagnetic wave travelling through vacuum has electric field $\vec{E}$ and magnetic field $\vec{B}$ defined as

$$\vec{E} = (\hat{\imath} + \hat{\jmath})E_0 \exp i(\omega t - \vec{\kappa} \cdot \vec{x}) \qquad \vec{B} = (\hat{\imath} - \hat{\jmath} - \hat{k})B_0 \exp i(\omega t - \vec{\kappa} \cdot \vec{x})$$

where E_0 and B_0 are real constants. The time-averaged Poynting vector will be given by

(a) $\vec{S} = -\frac{2}{\sqrt{\epsilon_0 \mu_0}} E_0 B_0 (\hat{\imath} - \hat{\jmath} + 2\hat{k})$

(b) $\vec{S} = -\frac{1}{2}\sqrt{\frac{3\epsilon_0}{\mu_0}} E_0^2 (\hat{\imath} - \hat{\jmath} + 2\hat{k})$

(c) $\vec{S} = \sqrt{\frac{\epsilon_0}{6\mu_0}} E_0^2 (-\hat{\imath} + \hat{\jmath} - 2\hat{k})$ ✓

(d) $\vec{S} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}} B_0^2 (\hat{\imath} - \hat{\jmath} + 2\hat{k})$

14. A thermally-insulated coffee mug contains 500 g of warm coffee at 80 °C. Assuming that the heat capacity of this liquid is 1 cal g^{-1} $°C^{-1}$ and the latent heat of fusion for ice is 80 cal g^{-1}, the amount of ice that must be dropped into the cup to convert it into cold coffee at 5 °C is about

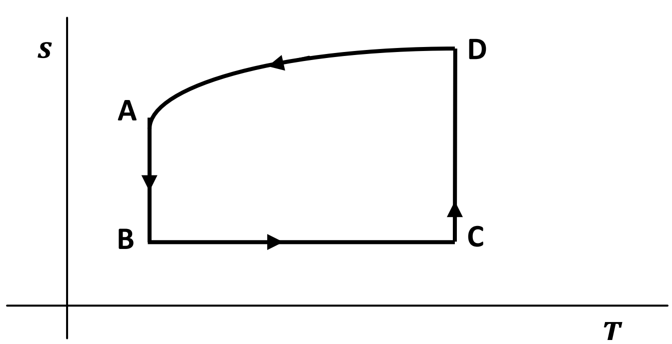
(a) 421 g

(b) 441 g ✓

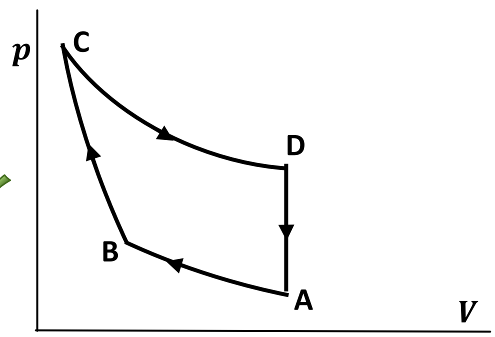
(c) 469 g

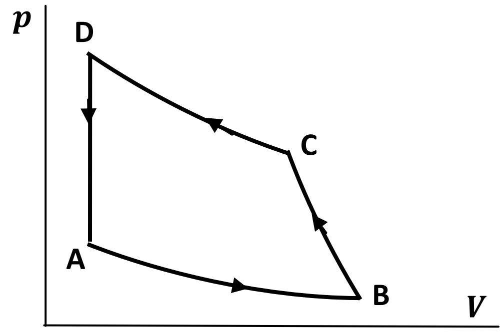
(d) 471 g

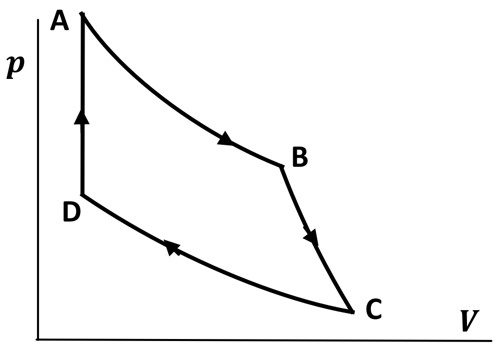
15. An ideal gas engine is run according to the cycle shown in the s-T diagram below, where the process from D to A is known to be isochoric (i.e. maintaining V = constant).

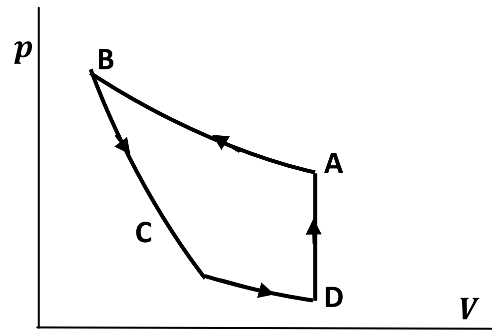


The corresponding cycle in the p-V diagram will most closely resemble

(a) 

(c) 

(b) 

(d) 

16. Consider a thermal ensemble at temperature T which is composed of identical quantum harmonic oscillators of frequency ω_0 with non-overlapping wavefunctions. The probability that there will be an even number of energy quanta in the system is

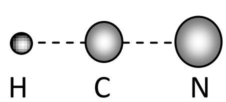
(a) $\dfrac{1}{\exp(-\hbar\omega_0/k_BT)+1}$ ✓

(c) $\dfrac{1}{\exp(-\hbar\omega_0/k_BT)-1}$

(b) $\dfrac{1}{\exp(\hbar\omega_0/k_BT)+1}$

(d) $\tanh(\hbar\omega_0/2k_BT)$

17. Consider the following linear model of a molecule of hydrogen cyanide (HCN) depicted below.



It follows that the molar specific heat of hydrogen cyanide gas at constant pressure must be

(a) 6 R (b) 4.5 R (c) 5 R (d) 5.5 R ✓

A

18. A beam of high energy neutrons is scattered from a metal lattice, where the spacing between nuclei is around 0.4 nm. In order to see quantum diffraction effects, the kinetic energy of the neutrons must be around

(a) 7.85 MeV (b) 5.11 meV ✓ (c) 511 keV (d) 78.5 eV

19. A particle of mass m, moving in one dimension, satisfies the modified Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + i\hbar u\frac{d\psi}{dx} = i\hbar\frac{d\psi}{dt}$$

where u is the velocity of the substrate. If, now, this particle is treated as a Gaussian wave packet peaked at wavenumber k, its group velocity will be $v_g =$

(a) $\frac{\hbar k}{2m} - u$

(b) $\frac{\hbar k}{m} + u$

(c) $\frac{\hbar k}{m} - u$ ✓

(d) $-\frac{\hbar k}{2m} + u$

20. In a one-dimensional system, the boundary condition that the derivative of the wavefunction $\psi'(x)$ should be continuous at every point is applicable whenever

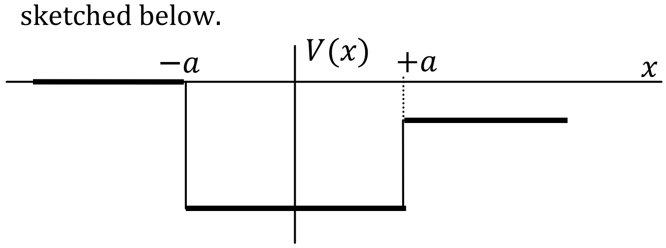
(a) the wavefunction $\psi(x)$ is itself continuous everywhere.

(b) there is a bound state and the potential is piecewise continuous.

(c) there is a bound state and the potential has no singularity anywhere. ✓

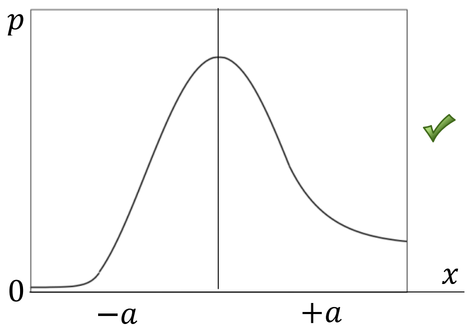
(d) there are bound or scattering states with definite momentum.

21. A particle moving in one dimension, is placed in an asymmetric square well potential $V(x)$ as sketched below.

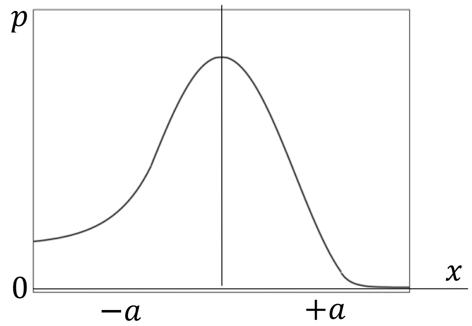


The probability density $p(x)$ in the ground state will most closely resemble

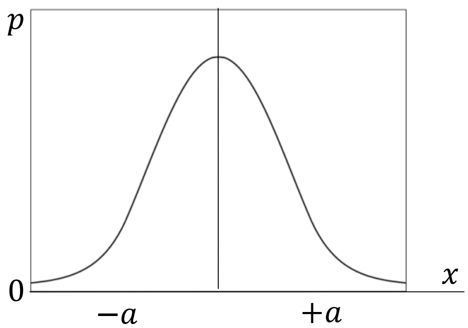
(a)



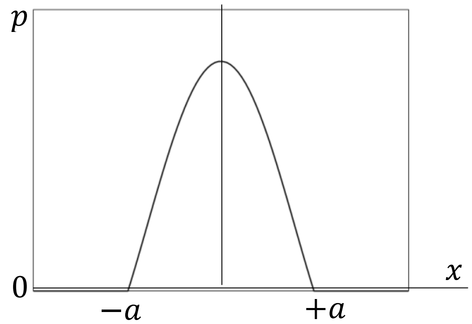
(c)



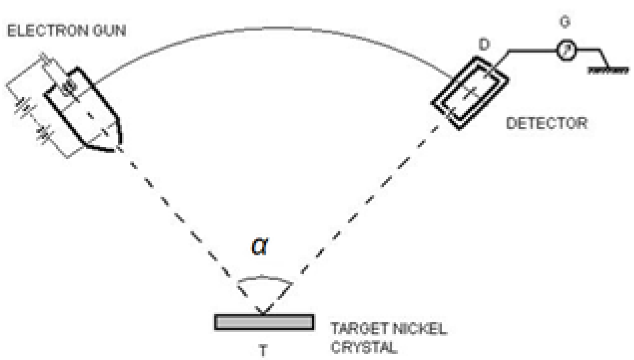
(b)

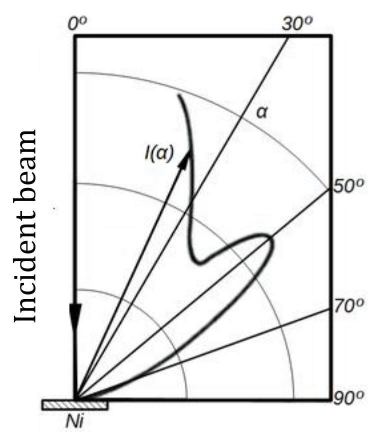


(d)



22. The sketch shown below illustrates the apparatus and results for a famous experiment. The graph on the right is a polar plot of the number of electrons received in the detector.





From the result, the experimenters were able to conclude that

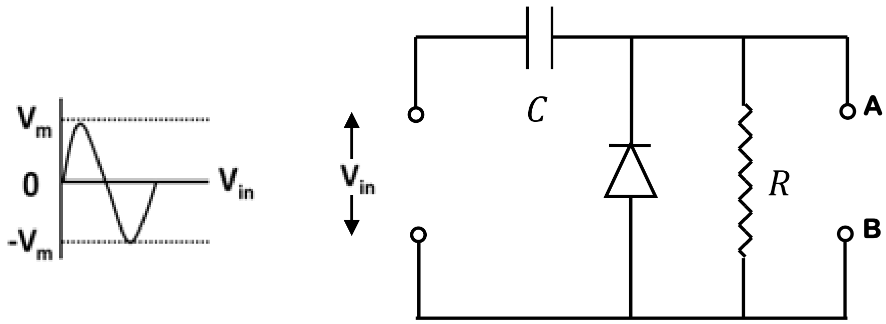
(a) the energy levels of atoms in a metal are quantized.

(b) electrons in a beam can behave as waves.

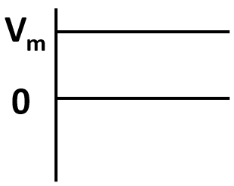
(c) electrons have spin half.

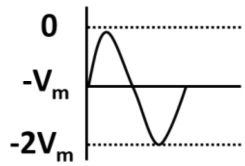
(d) there are magnetic domains inside a nickel sample.

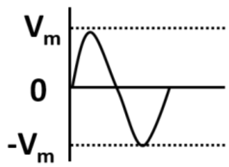
23. The signal shown on the left side of the figure below is fed into the circuit shown on the right side.

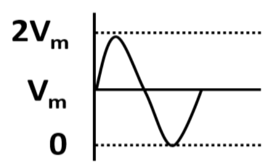


If the signal has time period τ_S and the circuit has a natural frequency τ_{RC}, then, in the case when $\tau_S \ll \tau_{RC}$, the steady-state output will resemble

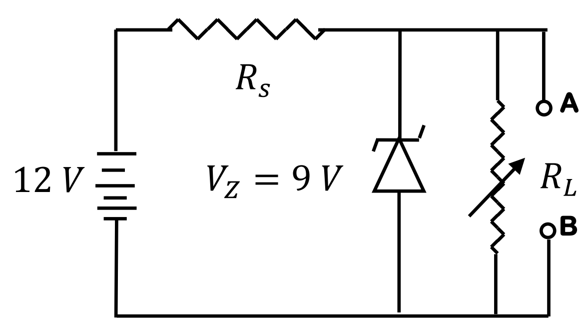
(a) 

(c) 

(b) 

(d) 

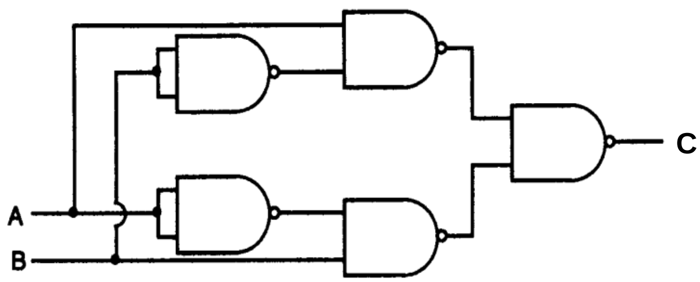
24. Drawing power from a 12 V car battery, a 9 V stabilized DC voltage is required to power a car stereo system, attached to the terminals A and B, as shown in the figure.



If a Zener diode with ratings, V_Z = 9 V and P_{max} = 0.27 W, is connected as shown in the figure, for the above purpose, the minimum series resistance R_s must be

(a) 111 Ω (b) 103 Ω (c) 100 Ω (d) 97 Ω

25. The circuit shown below uses only NAND gates.



The final output at **C** is

(a) **A** AND **B** (b) **A** OR **B** (c) **A** XOR **B** (d) **A** NOR **B**

Section B

This Section consists of 15 questions. All are of multiple-choice type. Mark only one option on the ORS provided to you. If more than one option is marked, it will be assumed that the question has not been attempted. A correct answer will get +5 marks, an incorrect answer will get 0 mark.

1. The integral

$$I = \int_0^\infty dx\ e^{-x}\ \delta(\sin x)$$

where $\delta(x)$ denotes the Dirac delta function, is

(a) 1

(b) $\dfrac{\exp \pi}{\exp \pi + 1}$

(c) $\dfrac{\exp \pi}{\exp \pi - 1}$ ✔

(d) $\dfrac{1}{\exp \pi - 1}$

2. Consider the complex function

$$f(x,y) = u(x,y) + iv(x,y)$$

where

$$u(x,y) = x^2(2+x) - y^2(2+3x)$$
$$v(x,y) = y(\lambda x + 3x^2 - y^2)$$

and λ is real. If it is known that $f(x,y)$ is analytic in the complex plane of $z = x + iy$, then it can be written

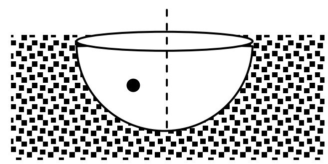
(a) $f = z^2(2+z)$ ✓

(b) $f = \bar{z}(2+\bar{z}^2)$

(c) $f = 2z\bar{z} + z^2 - \bar{z}^2$

(d) $f = z^2 + z^3$

3. A bead of mass m slides under the influence of gravity along the frictionless interior of a hemispherical cup of radius a sunk vertically into the ground with its open side level with the ground (see sketch on the right). In terms of spherical polar coordinates (θ, φ) set up with the centre of the upper circle as origin, the Hamiltonian H for this system will be



(a) $H = \dfrac{m}{2}\left(a^2\dot{\theta}^2 + a^2\,\dot{\varphi}^2\csc^2\theta\right) + 2mga\sin^2\dfrac{\theta}{2}$

(b) $H = \dfrac{m}{2}\left(a^2\dot{\theta}^2 + a^2\,\dot{\varphi}^2\sin^2\theta\right) + 2mga\sin^2\dfrac{\theta}{2}$

(c) $H = \dfrac{1}{2ma^2}\left(p_\theta^2 + p_\varphi^2\sin^2\theta\right) + 2mga\sin^2\dfrac{\theta}{2}$

(d) $H = \dfrac{1}{2ma^2}\left(p_\theta^2 + p_\varphi^2\csc^2\theta\right) + 2mga\sin^2\dfrac{\theta}{2}$ ✓

4. On a compact stellar object the gravity is so strong that a body falling from rest will soon acquire a velocity comparable with that of light. If the force on this body is $F = mg$ where m is the relativistic mass and g is a constant, the velocity of this falling body will vary with time as

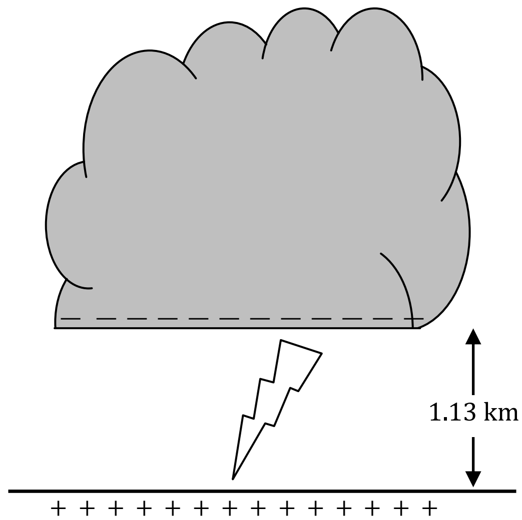
(a) $v = \dfrac{c}{1 - \dfrac{2c}{gt}}$

(b) $v = c\tanh\dfrac{gt}{c}$ ✓

(c) $v = \dfrac{2c}{\pi}\tan\dfrac{gt}{c}$

(d) $v = c\left\{1 - \exp\left(-\dfrac{gt}{c}\right)\right\}$

5. A monsoon cloud has a flat bottom of surface area 125 km^2. It floats horizontally over the ground at a level such that the base of the cloud is 1.13 km above the ground (see figure). Due to friction with the air below, the base of the cloud acquires a uniform electric charge density. This keeps increasing slowly with time.



When the uniform electric field below the cloud reaches the value 2.4 MV m^{-1} a lightning discharge occurs, and the entire charge of the cloud passes to the Earth below — which, in this case, behaves like a grounded conductor.

Neglecting edge effects and inhomogeneities inside the cloud and the air below, the energy released in this lightning discharge can be estimated, in kilowatt-hours (kWh), as about

(a) 10^9 ✓ (b) 10^5 (c) 10 (d) 10^{-1}

6. The magnetic vector potential corresponding to a uniform magnetic field $\vec{B}$ is often taken as

$$\vec{A} = \frac{1}{2}\vec{B} \times \vec{x}$$

This choice is

(a) valid in the Lorenz gauge.

(b) valid in the Coulomb gauge. ✓

(c) valid in the Weyl gauge.

(d) gauge invariant.

7. An ideal monatomic gas at chemical potential $\mu = -1$ eV and a temperature given by $k_B T = 0.1$ eV is in equilibrium with an adsorbing metal surface, i.e. there are isolated sites distributed randomly on the metal surface where the gas atom can get bound. Each such binding site can adsorb 0, 1, or 2 atoms with the released energy being 0, -1 eV and -1.9 eV respectively. The average number of adsorbed molecules at each site would be

(a) $\frac{1+e}{1-e}$ (b) $\frac{1+e}{1+2e}$ (c) $\frac{2+e}{1+2e}$ ✓ (d) $\frac{1+2e}{1+e}$

8. Consider N non-interacting distinguishable particles in equilibrium at an absolute temperature T. Each particle can only occupy one of two possible states of energy 0 and ϵ respectively ($\epsilon > 0$). The entropy of the system, in terms of $\beta = \epsilon/k_B T$ is

(a) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1+e^{-\beta}}\right\}$

(b) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) + \frac{\beta e^{-\beta}}{1+e^{-\beta}}\right\}$ ✓

(c) $Nk_B\left\{\ln\left(1-e^{-\beta}\right) + \frac{\beta e^{-\beta}}{1-e^{-\beta}}\right\}$

(d) $Nk_B\left\{\ln\left(1+e^{-\beta}\right) - \frac{e^{-\beta}}{1-e^{-\beta}}\right\}$

9. An electron in a hydrogen atom is in a state described by the wavefunction:

$$\Psi(\vec{r}) = \frac{1}{\sqrt{10}}\,\psi_{100}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{210}(\vec{x}) + \sqrt{\frac{2}{5}}\psi_{211}(\vec{x}) - \frac{1}{\sqrt{10}}\,\psi_{21,-1}(\vec{x})$$

where $\psi_{n\ell m}(\vec{x})$ denotes a normalized wavefunction of the hydrogen atom with the principal quantum number n, angular quantum number ℓ and magnetic quantum number m.

Neglecting the spin-orbit interaction, the expectation values of $\hat{L}_z$ and $\widehat{L^2}$ for this state are

(a) $3\hbar/_{10}, 9\hbar^2/_5$ ✓

(b) $3\hbar/_5, 9\hbar^2/_{10}$

(c) $3\hbar/_4, 9\hbar^2/_{25}$

(d) $8\hbar/_{10}, 3\hbar^2/_5$

10. A system of two spin-½ particles 1 and 2 has the Hamiltonian

$$\hat{H} = \epsilon_0\,\hat{h}_1 \otimes \hat{h}_2.$$

where

$$\hat{h}_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \hat{h}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

and ϵ_0 is a constant with the dimension of energy. The ground state of this system has energy

(a) $\sqrt{2}\epsilon_0$

(b) 0

(c) $-2\epsilon_0$ ✓

(d) $-4\epsilon_0$

11. At time $t = 0$, the wavefunction of a particle in a harmonic oscillator potential of natural frequency ω is given by

$$\psi(0) = \frac{1}{5}\left\{3\varphi_0 - 2\sqrt{2}\,\varphi_1 + 2\sqrt{2}\,\varphi_2\right\}$$

where $\varphi_n(x)$ denotes the eigenfunction belonging to the n-th eigenvalue of energy. At time $t = \tau$, the wavefunction is found to be

$$\psi(\tau) = -\frac{i}{5}\left\{3\varphi_0 + 2\sqrt{2}\,\varphi_1 + 2\sqrt{2}\,\varphi_2\right\}$$

The minimum value of τ is

(a) $\frac{\pi}{2\omega}$

(b) $\frac{2\pi}{\omega}$

(c) $\frac{2\pi}{3\omega}$

(d) $\frac{\pi}{\omega}$ ✓

12. At low temperatures, the measured specific heat C_V of a solid sample is found to depend on temperature as

$$C_V = aT^{3/2} + bT^3.$$

where a and b are constants. This material has

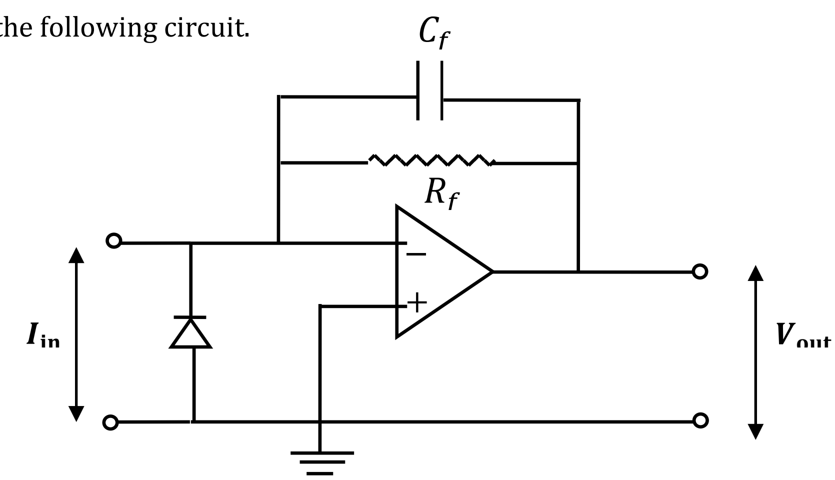
(a) one fermionic excitation with dispersion relation $\omega \propto k^4$;
another bosonic excitation with dispersion relation $\omega \propto k$

(b) one fermionic excitation with dispersion relation $\omega \propto k^2$;
another bosonic excitation with dispersion relation $\omega \propto k^4$

(c) one bosonic excitation with dispersion relation $\omega \propto k^2$; ✓
another bosonic excitation with dispersion relation $\omega \propto k$

(d) one fermionic excitation with dispersion relation $\omega \propto k^2$;
another fermionic excitation with dispersion relation $\omega \propto k$

13. Consider the following circuit.



It is given that $C_f = 100$ pF, and for $I_{in} = 50$ nA D.C., $V_{out} = 1$ V D.C. Therefore, the bandwidth of the above circuit is

(a) 15.8 Hz (b) 79.6 Hz ✓ (c) 145.3 Hz (d) 200.4Hz

14. The semi-empirical mass formula for a heavy nucleon (Z, A) can be written, to some approximation, as

$$M(Z,A)c^2 = ZM_pc^2 + (A-Z)M_nc^2 - \lambda_1 A - \lambda_2 A^{2/3} - \lambda_3 \frac{Z(Z-1)}{A^{1/3}} - \lambda_4 \frac{(A-2Z)^2}{A} - \frac{\lambda_5}{A^{1/2}}$$

where $M_pc^2 = 938$ MeV, $M_nc^2 = 939$ MeV, and

$$\lambda_1 = 16, \quad \lambda_2 = 18, \quad \lambda_3 = 0.7, \quad \lambda_4 = 23,$$

all in MeV, where

$$\lambda_5 = \begin{cases} +12 \text{ MeV for even} - \text{even nuclei} \\ -12 \text{ MeV for odd} - \text{odd nuclei} \\ 0 \quad \text{for others} \end{cases}$$

Now, consider a spontaneous fission reaction

$$^{238}_{92}\mathrm{U} \rightarrow {}^{146}_{56}\mathrm{Ba} + {}^{91}_{36}\mathrm{Kr} + {}^{1}_{0}\mathrm{n}$$

The energy released in this reaction will be close to

(a) 17.92 keV (b) 19.2 MeV (c) 170 MeV (d) 190 MeV

15. The table below gives the properties of four unstable particles $\mu^+, \pi^+, n^0, \Lambda^0$

Particle	Mass (MeV/c^2)	Spin	Principal decay mode
muon μ^+	105.66	½	$\mu^+ \rightarrow e^+ + \nu_\mu + \bar{\nu}_e$
pion π^+	139.57	0	$\pi^+ \rightarrow \mu^+ + \nu_\mu$
neutron n^0	939.56	½	$n^0 \rightarrow p^+ + e^- + \bar{\nu}_e$
Lambda hyperon Λ^0	1,115.68	½	$\Lambda^0 \rightarrow p^+ + \pi^-$

If arranged in order of DECREASING decay lifetime, the above list will read

(a) $n^0, \mu^+, \pi^+, \Lambda^0$ (b) $\mu^+, \Lambda^0, n^0, \pi^+$ (c) $n^0, \Lambda^0, \mu^+, \pi^+$ (d) $\pi^+, n^0, \mu^+, \Lambda^0$