Joint Entrance Screening Test (JEST-2015)-X

1. A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2 + y^2)$, under the action of gravity acting in the negative z-direction. $\vec{g} = -g\hat{z}$. What speed should he maintain to be able to ride at a constant height z_0 without failing down ?

(a) $\sqrt{gz_0}$

(b) $\sqrt{3gz_0}$

(c) $\sqrt{2gz_0}$

(d) The biker will not be able to maintain a constant height, irrespective of speed.

2. A chain of mass M and length L is suspended vertically with its lower end touching a weighing scale. The chain is released and falls freely onto the scale. Neglecting the size of the individual links, what is the reading of the scale when a length x of the chain has fallen ?

(a) $\frac{Mgx}{L}$ (b) $\frac{2Mgx}{L}$ (c) $\frac{3Mgx}{L}$ (d) $\frac{4Mgx}{L}$

3. The distance of a star from the Earth is 4.25 light years, as measured from the Earth. A space ship travels from Earth to the star at a constant velocity in 4.25 years, according to the clock on the ship. The speed of the space ship in units of the speed of light is,

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{2}{3}$ (d) $\frac{1}{\sqrt{3}}$

4. Consider a spin-$\frac{1}{2}$ particle characterized by the Hamiltonian $H = \omega S_z$. Under a perturbation $H' = gS_x$, the second order correction to the ground state energy is given by,

(a) $-\frac{g^2}{4\omega}$ (b) $\frac{g^2}{4\omega}$ (c) $-\frac{g^2}{2\omega}$ (d) $\frac{g^2}{2\omega}$

5. The Bernoulli polynomial $B_n(s)$ are defined by, $\frac{xe^{xs}}{e^x - 1} = \sum B_n(s)\frac{x^n}{n!}$. Which one of the following relations is true ?

(a) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)\frac{x^n}{(n+1)!}$

(b) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n\frac{x^n}{(n+1)!}$

(c) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(-s)(-1)^n\frac{x^n}{n!}$

(d) $\frac{xe^{x(1-s)}}{e^x - 1} = \sum B_n(s)(-1)^n\frac{x^n}{n!}$

6. What is the maximum number of extrema of the function $f(x) = P_k(x)e^{-\left(\frac{x^4}{4}+\frac{x^2}{2}\right)}$, where $x \in (-\infty, \infty)$ and $P_k(x)$ is an arbitrary polynomial of degree k ?

(a) $k + 2$ (b) $k + 6$ (c) $k + 3$ (d) k

7. Given an analytic function $f(z) = \phi(x, y) + i\psi(x, y)$, where $\phi(x, y) = x^2 + 4x - y^2 + 2y$. If C is constant, which of the following relations is true ?

(a) $\psi(x, y) = x^2 y + 4y + C$
(b) $\psi(x, y) = 2xy - 2x + C$
(c) $\psi(x, y) = 2xy + 4y - 2x + C$
(d) $\psi(x, y) = x^2 y - 2x + C$

8. A particle of mass m is confined in a potential well given by $V(x) = 0$ for $-L/2 < x < L/2$ and $V(x) = \infty$ elsewhere. A perturbing potential $H'(x) = \alpha x$ has been applied to the system. Let the first and second order corrections to the ground state be $E_0^{(1)}$ and $E_0^{(2)}$, respectively. Which one of the following statements is correct ?

(a) $E_0^{(1)} < 0$ and $E_0^{(2)} > 0$
(b) $E_0^{(1)} = 0$ and $E_0^{(2)} > 0$
(c) $E_0^{(1)} > 0$ and $E_0^{(2)} < 0$
(d) $E_0^{(1)} = 0$ and $E_0^{(2)} < 0$

9. Consider a harmonic oscillator in the state $|\psi\rangle = e^{-\frac{|\alpha|^2}{2}} e^{\alpha a^\dagger} |0\rangle$, where $|0\rangle$ is the ground state, $a^\dagger$ is the raising operator and α is a complex number. What is the probability that the harmonic oscillator is in the n-th eigenstate $|n\rangle$?

(a) $e^{-|\alpha|^2} \dfrac{|\alpha|^{2n}}{n!}$
(b) $e^{-\frac{|\alpha|^2}{2}} \dfrac{|\alpha|^n}{\sqrt{n!}}$
(c) $e^{-|\alpha|^2} \dfrac{|\alpha|^n}{n!}$
(d) $e^{-\frac{|\alpha|^2}{2}} \dfrac{|\alpha|^{2n}}{n!}$

10. A particle of mass m moves in 1-dimensional potential $V(x)$, which vanishes at infinity. The exact ground state eigenfunction is $\psi(x) = A\,\text{sech}(\lambda x)$ where A and λ are constant. The ground state energy eigenvalue of this system is,

(a) $E = \dfrac{\hbar^2 \lambda^2}{m}$
(b) $E = -\dfrac{\hbar^2 \lambda^2}{m}$
(c) $E = -\dfrac{\hbar^2 \lambda^2}{2m}$
(d) $E = \dfrac{\hbar^2 \lambda^2}{2m}$

11. Given that ψ_1 and ψ_2 are eigenstates of a Hamiltonian with eigenvalues E_1 and E_2 respectively, what is the energy uncertainty in the state $(\psi_1 + \psi_2)$?

(a) $-\sqrt{E_1 E_2}$
(b) $\dfrac{1}{2}|E_1 - E_2|$
(c) $\dfrac{1}{2}|E_1 + E_2|$
(d) $\dfrac{1}{\sqrt{2}}|E_2 - E_1|$

12. Consider two points charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum ?

(a) μ
(b) 1
(c) $\dfrac{1}{\mu}$
(d) $1 + \mu$

13. If two ideal dice are rolled once, what is the probability of getting at least one '6' ?

(a) $\dfrac{11}{36}$
(b) $\dfrac{1}{36}$
(c) $\dfrac{10}{36}$
(d) $\dfrac{5}{36}$

14. An ideal gas is compressed adiabatically from an initial volume V to a final volume αV and a work W is done on the system in doing so. The final pressure of the gas will be $\left(\gamma = \dfrac{C_P}{C_V}\right)$,

(a) $\dfrac{W}{V^\gamma} \dfrac{1-\gamma}{\alpha - \alpha^\gamma}$
(b) $\dfrac{W}{V^\gamma} \dfrac{\gamma - 1}{\alpha - \alpha^\gamma}$
(c) $\dfrac{W}{V} \dfrac{1-\gamma}{\alpha - \alpha^\gamma}$
(d) $\dfrac{W}{V} \dfrac{\gamma - 1}{\alpha - \alpha^\gamma}$

15. For a system in thermal equilibrium with a heat bath at temperature T, which one of the following equalities is correct ?

$$\left(\beta = \frac{1}{k_B T}\right)$$

(a) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E\rangle^2 - \langle E^2\rangle$ (b) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E^2\rangle - \langle E\rangle^2$

(c) $\frac{\partial}{\partial\beta}\langle E\rangle = \langle E^2\rangle + \langle E\rangle^2$ (d) $\frac{\partial}{\partial\beta}\langle E\rangle = -\left(\langle E^2\rangle + \langle E\rangle^2\right)$

16. A spherical shell of inner and outer radii a and b, respectively, is made of a dielectric material with frozen polarization $\vec{P}(r) = \frac{k}{r}\hat{r}$, where k is a constant and r is the distance from the its centre. The electric field in the region $a < r < b$ is ,

(a) $\vec{E} = \frac{k}{\varepsilon_0 r}\hat{r}$ (b) $\vec{E} = -\frac{k}{\varepsilon_0 r}\hat{r}$ (c) $\vec{E} = 0$ (d) $\vec{E} = \frac{k}{\varepsilon_0 r^2}\hat{r}$

17. The electrostatic potential due to a change distribution is given by $V(r) = A\frac{e^{-\lambda r}}{r}$, where A and λ are constants. The total charge enclosed within a sphere of radius $1/\lambda$, with its origin at $r = 0$ is given by,

(a) $\frac{8\pi\varepsilon_0 A}{e}$ (b) $\frac{4\pi\varepsilon_0 A}{e}$ (c) $\frac{\pi\varepsilon_0 A}{e}$ (d) 0

18. A circular loop of radius R, carries a uniform line charge density λ. The electric field, calculated at a distance z directly above the center of the loop, is maximum if z is equal to,

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2}$ (d) $2R$

19. For non-interacting Fermions in d-dimensions, the density of states $D(E)$ varies as $E^{(d/2-1)}$. The Fermi energy E_F of an N particle system in 3-, 2- and 1-dimensions will scale respectively as,

(a) $N^2, N^{2/3}, N$ (b) $N, N^{2/3}, N^2$ (c) $N, N^2, N^{2/3}$ (d) $N^{2/3}, N, N^2$

20. A classical particle with total energy E moves under the influence of a potential $V(x, y) = 3x^3 + 2x^2 y + 2xy^2 + y^3$. The average potential energy, calculated over a long time is equal to,

(a) $\frac{2E}{3}$ (b) $\frac{E}{3}$ (c) $\frac{E}{5}$ (d) $\frac{2E}{5}$

21. A particle in thermal equilibrium has only 3 possible states with energies $-\varepsilon, 0, \varepsilon$. If the system is maintained at a temperature $T \gg \frac{\varepsilon}{k_B}$, then the average energy of the particle can be approximated to,

(a) $\frac{2\varepsilon^2}{3k_B T}$ (b) $-\frac{2\varepsilon^2}{3k_B T}$ (c) $-\frac{\varepsilon^2}{k_B T}$ (d) 0

22. The energy difference between the $3p$ and $3s$ levels in Na is 2.1 eV. Spin-orbit coupling splits the $3p$ level, resulting in two emission lines differing by 6 Å. The splitting of the $3p$ level is approximately,

(a) 2 eV (b) 0.2 eV (c) 0.02 eV (d) 2 meV

23. What is the voltage at the output of the following operational amplifier circuit [see figure 1] ?

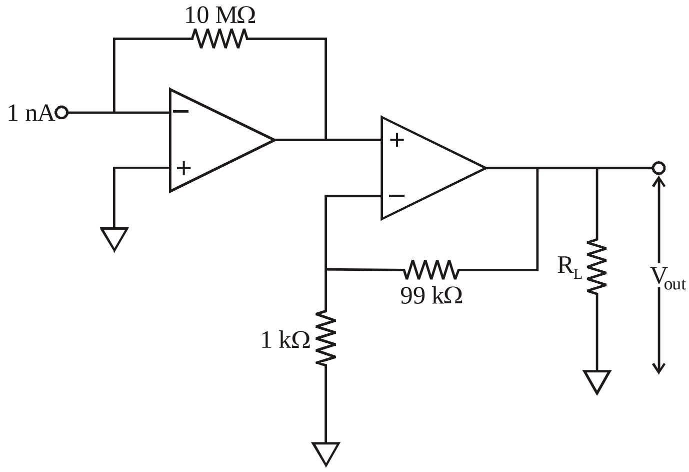

Figure 1

(a) 1 V (b) 1 mV (c) 1 μV (d) 1 nV

24. What is the area of the irreducible Brillouin zone of the crystal structure as given in the figure ? [see figure 2]

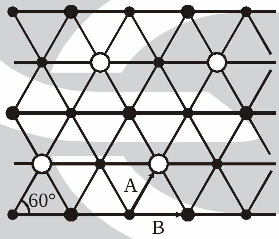

Figure 2

(a) $\frac{2\pi^2}{\sqrt{3}A^2}$ (b) $\frac{\sqrt{3}\pi^2}{2A^2}$ (c) $\frac{2\pi^2}{A^2}$ (d) $\frac{\pi^2}{\sqrt{3}A^2}$

25. For a 2-dimensional honeycomb lattice as shown in the figure 3, the first Bragg spot occurrs for the grazing angle θ_1, while sweeping the angle from 0°. The next Bragg spot is obtained at θ_2 given by,

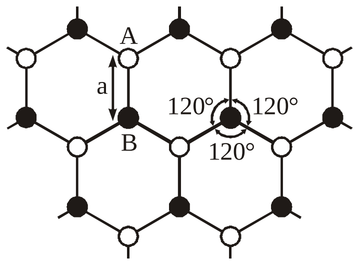

Figure 3

(a) $\sin^{-1}(3\sin\theta_1)$ (b) $\sin^{-1}\left(\frac{3}{2}\sin\theta_1\right)$ (c) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta_1\right)$ (d) $\sin^{-1}\left(\sqrt{3}\sin\theta_1\right)$

PART-B : 1 MARK QUESTIONS

26. How is your weight affected if the Earth suddenly doubles in radius, mass remaining the same ?
(a) Increases by a factor of 4 (b) Increases by a factor of 2
(c) Decreases by a factor of 4 (d) Decreases by a factor of 2

27. A spring of force constant k is stretched by x. It takes twice as much work to stretch a second spring by $\frac{x}{2}$. The force constant of the second spring is,
(a) k (b) $2k$ (c) $4k$ (d) $8k$

28. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true ?
(a) This is a free particle
(b) The particle is experiencing velocity dependent damping
(c) The particle is executing simple harmonic motion
(d) The particle is under constant acceleration

29. A charged particle is released at time $t = 0$, from the origin in the presence of uniform static electric and magnetic fields given by $\mathbf{E} = E_0\hat{y}$ and $\mathbf{B} = B_0\hat{z}$, respectively. Which of the following statements is true for $t > 0$?
(a) The particle moves along the x-axis (b) The particle moves in a circular orbit
(c) The particle moves in the (x, y) plane (d) The particle moves in the (y, z) plane

30. The sum $\sum_{M=1}^{99} \frac{1}{\sqrt{m+1}+\sqrt{m}}$ is equal to
(a) 9 (b) $\sqrt{99}-1$ (c) $\frac{1}{\left(\sqrt{99}-1\right)}$ (d) 11

31. Which of the following expressions represents an electric field due to a time varying magnetic field ?
(a) $K(x\hat{x} + y\hat{y} + z\hat{z})$ (b) $K(x\hat{x} + y\hat{y} - z\hat{z})$
(c) $K(x\hat{x} - y\hat{y})$ (d) $K(y\hat{y} - x\hat{y} + 2z\hat{z})$

32. The skin depth of a metal is dependent on the conductivity (σ) of the metal and the angular frequency ω of the incident field. For a metal of high conductivity, which of the following relations is correct ? (Assume that $\sigma \gg \varepsilon\omega_1$ where ε is the electrical permittivity of the medium).
(a) $d \propto \sqrt{\sigma/\omega}$ (b) $d \propto \sqrt{1/\sigma\omega}$ (c) $d \propto \sqrt{\sigma\omega}$ (d) $d \propto \sqrt{\omega/\sigma}$

33. Consider the differential equation $G'(x) + kG(x) = \delta(x)$; where k is a constant. Which of the following statements is true ?
(a) Both $G(x)$ and $G'(x)$ are continuous at $x = 0$
(b) $G(x)$ is continuous at $x = 0$ but $G'(x)$ is not
(c) $G(x)$ is discontinuous at $x = 0$
(d) The continuity properties of $G(x)$ and $G'(x)$ at $x = 0$ depends on the value of k

34. A particle moving under the influence of a potential $V(r) = kr^2/2$ has a wavefunction $\psi(r, t)$. If the wavefunction changes to $\psi(\alpha r, t)$, the ratio of the average final kinetic energy to the initial kinetic energy will be,
(a) $\frac{1}{\alpha^2}$ (b) α (c) $\frac{1}{\alpha}$ (d) α^2

35. If a Hamiltonian H is given as $H = |0\rangle\langle 0| - |1\rangle\langle 1| + i\left(|0\rangle\langle 1| - |1\rangle\langle 0|\right)$, where $|0\rangle$ and $|1\rangle$ are orthonormal states, the eigenvalues of H are

(a) ± 1 (b) $\pm i$ (c) $\pm\sqrt{2}$ (d) $\pm i\sqrt{2}$

36. Electrons of mass m in a thin, long wire at a temperature T follow a one-dimensional Maxwellian velocity distribution. The most probable speed of these electrons is,

(a) $\sqrt{\left(\frac{kT}{2\pi m}\right)}$ (b) $\sqrt{\left(\frac{2kT}{m}\right)}$ (c) 0 (d) $\sqrt{\left(\frac{8kT}{\pi m}\right)}$

37. The blackbody at a temperature of 6000 K emits a radiation whose intensity spectrum peaks at 600 nm. If the temperature is reduced to 300 K, the spectrum will peak at,

(a) 120 μm (b) 12 μm (c) 12 mm (d) 120 mm

38. Let λ be the wavelength of incident light. The diffraction pattern of a circular aperture of dimension r_0 will transform from Fresnel to Fraunhofer regime if the screen distance z is,

(a) $z \gg r_0^2/\lambda$ (b) $z \gg \lambda^2/r_0$ (c) $z \ll \lambda^2/r_0$ (d) $z \ll r_0^2/\lambda$

39. The approximate force exerted on a perfectly reflecting mirror by an incident laser beam of power 10 mW at normal incidence is

(a) 10^{-13} N (b) 10^{-11} N (c) 10^{-9} N (d) 10^{-15} N

40. The wavelength of red helium-neon laser in air is 6328 Å. What happens to its frequency in glass that has a refractive index of 1.50 ?

(a) Increases by a factor of 3 (b) Decreases by a factor of 1.5
(c) Remains the same (d) Decreases by a factor of 0.5

41. The reaction $e^+ + e^- \to \gamma$ is forbidden because,

(a) lepton number is not conserved (b) linear momentum is not conserved
(c) angular momentum is not conserved (d) charge is not conserved

42. In Millikan's oil drop experiment the electronic charge e could be written as $\kappa\eta^{1.5}$, where κ is a function of all experimental parameters with negligible error. If the viscosity of air η is taken to be 0.4% lower than the actual value, what would be the error in the calculated in the calculated value of e ?

(a) 1.5% (b) 0.7% (c) 0.6% (d) 0.4%

43. The stable nucleus that has $\frac{1}{3}$ the radius of ^{189}Os nucleus is,

(a) ^{7}Li (b) ^{16}O (c) ^{4}He (d) ^{14}N

44. Which of the following excited states of a hydrogen atom has the highest lifetime ?

(a) $2p$ (b) $2s$ (c) $3s$ (d) $3p$

45. Which of the following statements is true for the energies of the terms of the carbon atom in the ground state electronic configuration $1s^2 2s^2 2p^2$?

(a) $^3P < {}^1D < {}^1S$ (b) $^3P < {}^1S < {}^1D$ (c) $^3P < {}^1F < {}^1S$ (d) $^3P < {}^1F < {}^1D$

46. For the logic circuit shown in figure 4, the required input condition (A, B, C) to make the output (X) = 1 is,

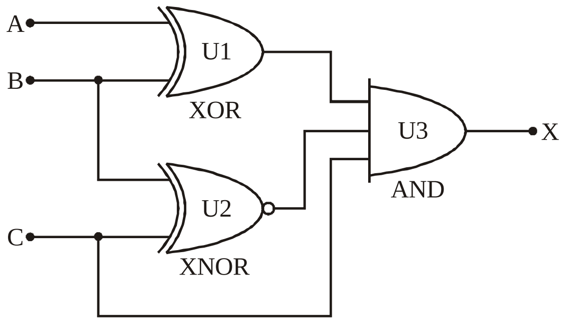

Figure 4

(a) 1, 0, 1 (b) 0, 0, 1 (c) 1, 1, 1 (d) 0, 1, 1

47. The reference voltage of an analog to digital converter is 1 V. The smallest voltage step that the converter can record using a 12-bit converter is,

(a) 0.24 V (b) 0.24 mV (c) 0.24 μV (d) 0.24 nV

48. The total number of Na^+ and Cl^- ions per unit cell of NaCl is,

(a) 2 (b) 4 (c) 6 (d) 8

49. The entropy temperature diagram of two Carnot engines, *A* and *B*, are shown in the figure 5. The efficiencies of the engines are η_A and η_B, respectively. Which one of the following equalities is correct ?

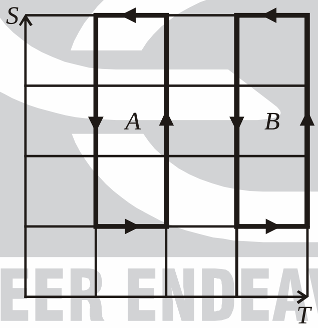

Figure 5

(a) $\eta_A = \eta_B/2$ (b) $\eta_A = \eta_B$ (c) $\eta_A = 3\eta_B$ (d) $\eta_A = 2\eta_B$

50. Given that tight binding dispersion relation $E(k) = E_0 + A\sin^2\left(\frac{ka}{2}\right)$, where E_0 and A are constants and a is the lattice parameter. What is the group velocity of an electron at the second Brillouin zone boundary?

(a) 0 (b) $\frac{a}{h}$ (c) $\frac{2a}{h}$ (d) $\frac{a}{2h}$