SECTION-A

1. ***This question consists of TWENTY FIVE sub-questions (1.1 to 1.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×1 = 25]**

1.1. If S is the closed surface enclosing a volume V and $\hat{n}$ is the unit normal vector to the surface and $\vec{r}$ is the position vector, then the value of the following integral $\iint_s \vec{r}\cdot\hat{n}dS$ is

(a) V (b) 2V (c) 0 (d) 3V

1.2. For any operator A, $i(A^+ - A)$ is

(a) Hermitian (b) anti-Hermitian (c) unitary (d) orthogonal

1.3. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is

(a) 0 (b) $z^{11}/11$ (c) $2\pi iz^{11}/11$ (d) 1/11

1.4. Consider the set of vectors $\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

(a) The three vectors are orthonormal
(b) The three vectors are linearly independent
(c) The three vectors cannot form a basis in a three-dimensional real vector space
(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as a linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

1.5. The Lagrangian for the Kepler problem is given by

$$L = \frac{1}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{\mu}{r}, (\mu > 0),$$

where (r,θ) denote the polar coordinates and the mass of the particle is unity. Then

(a) $p_\theta = 2r^2\dot{\theta}$ (b) $p_r = 2\dot{r}$
(c) the angular momentum of the particle about the centre of attraction is a constant
(d) the total energy of the particle is time dependent

1.6 Which of the following equations is relativistically invariant? (α, β, γ and δ are constants of suitable dimensions)

(a) $\frac{\partial\phi(x,t)}{\partial t} = \alpha\frac{\partial^2\phi(x,t)}{\partial x^2}$ (b) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \beta\frac{\partial^2\phi(x,t)}{\partial x^2}$

(c) $\frac{\partial^2\phi(x,t)}{\partial t^2} = \gamma\frac{\partial\phi(x,t)}{\partial x}$ (d) $\frac{\partial\phi(x,t)}{\partial t} = \delta\frac{\partial^3\phi(x,t)}{\partial x^3}$

1.7. The Lagrangian for a three particles system is given by:

$$L = \frac{1}{2}\left(\dot{\eta}_1^2 + \dot{\eta}_2^2 + \dot{\eta}_3^2\right) - a^2(\eta_1^2 + \eta_2^2 + \eta_3^2 - \eta_1\eta_3),$$

where a is real. Then one of the normal coordinates has a frequency ω given by

(a) $\omega^2 = a^2$ (b) $\omega^2 = a^2/2$ (c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}a^2$

1.8. Two point charges Q_1 = 1nC and Q_2 = 2nC are kept in free space such that the distance between them is 0.1 m.

(a) The force on Q_2 is along the direction from Q_2 to Q_1

(b) The force on Q_2 is the same in magnitude as that on Q_1

(c) The force on Q_1 is attractive

(d) A point charge Q_3 = –3nC, placed at the midpoint between Q_1 and Q_2, experiences no net force

1.9. A current I flows in the anticlockwise direction through a square loop of side a lying in the xoy plane with its centre at the origin. The magnetic induction at the centre of the square loop is

(a) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_x$ (b) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_z$ (c) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_z$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_x$

1.10. A thin conducting wire is bent into a circular loop of radius r and placed in a time dependent magnetic field of magnetic induction.

$$\vec{B}(t) = B_0 e^{-\alpha t}\hat{e}_z, (B_0 > 0 \text{ and } \alpha > 0),$$

such that, the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is

(a) $\pi r^2 \alpha B_0 e^{-\alpha t}$ (b) $\pi r^2 B_0 e^{-\alpha t}$ (c) $-\pi r^2 \alpha B_0 e^{-\alpha t}$ (d) $-\pi r^2 B_0 e^{-\alpha t}$

1.11. Consider an electric field $\vec{E}$ existing in the interface between a conductor and free space. Then the electric field $\vec{E}$ is

(a) external to the conductor and normal to the conductor's surface

(b) internal to the conductor and normal to the conductor's surface

(c) external to the conductor and tangential to the conductor's surface

(d) both external and internal to the conductor and normal to the conductor's surface

1.12. A spinless particle moves in a central potential V(r)

(a) The kinetic energy and the potential energy of the particle cannot simultaneously have sharp values

(b) The total energy and the potential energy of the particle can simultaneously have sharp values

(c) The total energy and the square of the orbital angular momentum about the origin cannot simultaneously have sharp values.

(d) The total energy of the particle can have only discrete eigenvalues

1.13. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$.

(a) $\varphi(x) = A\tan x, A > 0$ (b) $\varphi(x) = B\cos x, B$ real

(c) $\varphi(x) = C\exp(-D/x^2), C > 0, D < 0$ (d) $\varphi(x) = E\,x\exp(-Fx^2), E, F > 0$

1.14. A quantum harmonic oscillator is in the energy eigenstate $|n\rangle$. A time independent perturbation $\lambda(a^t a)^2$ acts on the particle, where λ is a constant of suitable dimensions and a and $a^\dagger$ are lowering and raising operators respectively. Then the first order energy shift is given by

(a) λn (b) $\lambda^2 n$ (c) λn^2 (d) $(\lambda n)^2$

1.15. Two particles are said to be distinguishable when

(a) the average distance between them is large compared to their de Broglie wavelengths

(b) the average distance between them is small compared to their de Broglie wavelengths

(c) they have overlapping wavepackets

(d) their total wave function is symmetric under particle exchange

1.16. For an energy state E of a photon gas, the density of states is proportional to

(a) $\sqrt{E}$ (b) E (c) $E^{3/2}$ (d) E^2

1.17. X-rays were produced using Cobalt (Z = 27) as target. It was observed that the X-ray spectrum contained a strong K_α line of wavelength 0.1785 nm and a weak K_α line of wavelength 0.1930 nm. Then, the weak K_α line is due to an impurity whose atomic number is

(a) 25 (b) 26 (c) 28 (d) 30

1.18. A sample of Silicon of thickness $200\mu m$ is doped with 10^{23} Phosphorous atoms per m^3. If the sample is kept in a magnetic field of 0.2 Wb/m^2 and a current of 1 mA is passed through the sample, the Hall voltage produced is

(a) $62.5\mu V$ (b) $-6.25\mu V$ (c) $+6.25\mu V$ (d) $-62.5\mu V$

1.19. The probability that a state which is 0.2 eV above the Fermi energy in a metal atom at 700K is

(a) 96.2% (b) 62.3% (c) 3.5% (d) 37.7%

1.20. The distance between the adjacent atomic planes in $CaCO_3$ is 0.3 nM. The smallest angle of Bragg scattering for 0.03 nm X-ray is

(a) 2.9º (b) 1.5º (c) 0.29º (d) 5.8º

1.21. Infrared absorption can be observed in which of the following molecules?

(a) N_2 (b) O_2 (c) HCl (d) C_2

1.22. The cross-sections of the reactions $p+\Pi^- \to \Sigma^- + K^+$ and $p^- + \Pi^+ \to \overline{\Sigma}^- + K^-$ at a given energy are the same due to

(a) baryon number conservation (b) time-reversal invariance

(c) charge conjugation (d) parity conservation

1.23. RAM and ROM are

(a) charge coupled devices used in computers (b) computer memories

(c) logic gates (d) binary counters used in computers

1.24. In an *n-p-n* transistor, the leakage current consists of

(a) electrons moving from the base to the emitter

(b) electrons moving from the collector to the base

(c) electrons moving from the collector to the emitter

(d) electrons moving from the base to the collector

1.25. A piece of semiconducting material is introduced into a circuit. If the temperature of the material is raised, the circuit current will

(a) increase (b) remain the same (c) decrease (d) cease to flow

2. ***This question consists of TWENTY FIVE sub-questions (2.1 to 2.25) of ONE mark each. For each of these sub-questions, four possible answers (a, b, c and d) are given, out of which only one is correct.*** **[25×2 = 50]**

2.1. If $\vec{A} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals

(a) 1 (b) 3 (c) 0 (d) –3

2.2. The value of the residue of $\frac{\sin z}{z^6}$ is

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$ (c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

2.3. If $F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$, then $\hat{F}^2[f(x)]$ is equal to

(a) $f(x)$ (b) $-f(x)$ (c) $f(-x)$ (d) $[f(x) + f(-x)]/2$

2.4. A particle of mass m is constrained to move on the plane curve $xy = C (C > 0)$ under gravity (y axis vertical). The Lagrangian of the particle is given by

(a) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C^2}{x^4}\right)+\frac{mgC}{x}$ (b) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C^2}{x^4}\right)-\frac{mgC}{x}$

(c) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C}{x^2}\right)+\frac{mgC}{x}$ (d) $\frac{1}{2}\dot{m}^2 x\left(1+\frac{C}{x^2}\right)-\frac{mgC}{x}$

2.5. If poisson bracket $[q, f(p)] = \alpha f(p)$, where α is a scalar then $f(p)$ is equal to

(a) $e^{\alpha p}$ (b) $e^{-\alpha p}$ (c) $\alpha e^{-\alpha p}$ (d) αe^{-p}

2.6. x and p are two operators which satisfy $[x, p] = i$. The operators X and P are defined as $X = x\cos\phi + p\sin\phi$ and $Y = -x\sin\phi + p\cos\phi$, for ϕ real. Then $[X, Y]$ equals

(a) 1 (b) –1 (c) i (d) $-i$

2.7. A quantum particle of mass m is confined to a square region in xoy-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for l, m, n positive integers)?

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$ (b) $\frac{2}{L}\cos\left(\frac{l\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$ (d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{l\pi y}{L}\right)$

2.8. Let $\vec{L} = (L_x, L_y, L_z)$ denote the orbital angular momentum operators of a particle and let $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigenstate of L^2 and L_z eigenvalues $\hbar^2\ell(\ell+1)$ and $\hbar\ell$ respectively. The expectation value of $L_+ L_-$ in this state is

(a) $\ell\hbar^2$ (b) $2\ell\hbar^2$ (c) 0 (d) $\ell\hbar$

2.9. A normalized state of a particle moving in a potential V(x) is given by $\psi(x,t) = \sum_{n=0}^{\infty} C_n \phi_n(x) e^{-E_n t/h}$

where $\phi_n(x)$'s are the normalized eigenfunctions of the particle corresponding to the energies E_n's. Then

(a) $\sum_{n=0} |C_n|^2 = 1$

(b) The average energy of the particle in the state $\psi(x,t)$ is $\sum_{n=\infty}^{\infty} |C_n| E_n$

(c) $\psi(x,t)$ is an eigenfunction of the Hamiltonian of the particle

(d) $\psi(x,t)$ is an eigenfunction of the momentum operator

2.10. A coaxial cable of uniform cross-section contains an insulating material of dielectric constant 3.5. The radius of the central wire is 0.01 m and that of the sheath is 0.02 m. The capacitance per kilometer of a cable is

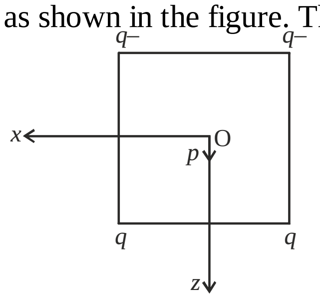
(a) 280.5 nF (b) 28.05 nF (c) 56.10 nF (d) 2.805 nF

2.11. The *xoy* plane carries a uniform surface current of density $\vec{K} = 50\hat{e}_z$ A/m. The magnetic field at the point $z = -0.5\text{m}$ is

(a) 10×10^{-6} Wb (b) 1×10^{-6} Wb (c) $\pi\times10^{-6}$ Wb (d) $10\pi\times10^{-6}$ Wb

2.12. Four point charges are placed at the corners of a square whose center is at the origin of a Cartesian coordinate system. A point dipole $\vec{p}$ is placed at the centre of the square as shown in the figure. Then,

(a) there is no force acting on the dipole
(b) there is no torque about the centre of O on the dipole
(c) the dipole has minimum energy if it is in $\hat{e}_x$ direction
(d) the force on the dipole is increased if the medium is replaced by another medium with larger dielectric constant



2.13. The electric field $E(r,t)$ at a point *r* at time *t* in a metal due to the passage of electrons can be described by the equation

$$\nabla^2\vec{E}(\vec{r},t) = \frac{1}{c^2}\left[\frac{\partial^2\vec{E}(\vec{r},t)}{\partial t^2} + \omega'^2\vec{E}(\vec{r},t)\right]$$

where ω' is a characteristic associated with the metal and *c* is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp\left[i(\vec{k}\cdot\vec{r} - \omega t)\right]$ is given by

(a) $\omega^2 = c^2k^2 - \omega'^2$ (b) $\omega^2 = c^2k^2 + \omega'^2$ (c) $\omega = ck - \omega'$ (d) $\omega = ck + \omega'$

2.14. A copper wire of uniform cross-sectional area 1.0×10^{-6} m^2 carries a current of 1A. Assuming that each copper atom contributes one electron to the electron gas, the drift velocity of the free electrons (density of copper is 8.94×10^3 kg/m^3 and its atomic mass is 1.05×10^{-25} kg) is

(a) 7.4×10^{-4} m/s (b) 74×10^{-4} m/s (c) 74×10^{-3} m/s (d) 7.4×10^{-5} m/s

2.15. The number of hyperfine components observed in the electronic transition ${}^2p_{1/2} \to {}^2S_{1/2}$ of an atom with nuclear spin $\frac{1}{2}$ is

(a) 3 (b) 4 (c) 6 (d) 5

2.16. Which of the following functions describes the nature of interaction potential V(r) between two quarks inside a nucleon? (*r* is the distance between the quarks and *a* and *b* positive constants of suitable dimensions)

(a) $V(r) = \frac{a}{r} + br$ (b) $V(r) = -\frac{a}{r} + br$ (c) $V(r) = \frac{a}{r} - br$ (d) $V(r) = -\frac{a}{r} - br$

2.17. Which of the following reactions violates lepton number conservation?

(a) $e^+ + e^- \to \upsilon + \bar{\upsilon}$ (b) $e^- + p \to \upsilon + n$

(c) $e^+ + n \to p + \upsilon$ (d) $\mu^- \to e^- + \upsilon + \bar{\upsilon}$

2.18. The Lande g-factor for the $^3P_1$ level of an atom is

(a) 1/2 (b) 3/2 (c) 5/2 (d) 7/2

2.19. The pure rotational levels of a molecule in the far-infrared region follows the formula F(J) = BJ (J+1), where F(J) is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is

(a) 2B (b) 4B (c) 6B (d) 8B

2.20. The total number of Zeeman components observed in an electronic transition $^2D_{5/2} \to {}^2P_{3/2}$ of an atom in a weak field is

(a) 4 (b) 6 (c) 12 (d) 10

2.21. A resistance of 600Ω is parallel to an inductance of reactance $600\,(\Omega)$ applied voltage, then the total impedance of the circuit is

(a) 628Ω (b) 268Ω (c) $424\,\Omega$ (d) 300Ω

2.22. An n-channel silicon (dielectric constant = 12) FET with a channel width $a = 2 \times 10^{-6}$ m is doped with 10^{21} electrons / m^3. The pinch-off voltage is

(a) 0.86V (b) 0.68V (c) 8.6V (d) 6.8V

2.23. The solution of the system of differential equations

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

(a) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} + 2Be^{-x}$ (b) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} + 2Be^{-x}$

(c) $y(x) = Ae^{3x} + Be^{-x}; z(x) = 2Ae^{3x} - 2Be^{-x}$ (d) $y(x) = Ae^{3x} + Be^{-x}; z(x) = -2Ae^{3x} - 2Be^{-x}$

2.24. If $u(x,y,z,t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave function

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} \text{ then } \beta \text{ is}$$

(a) $\left(1 - \frac{v}{c}\right)^{1/2}$ (b) $\left(1 - \frac{v}{c}\right)$ (c) $\left(1 - \frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1 - \frac{v^2}{c^2}\right)$

2.25. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by

(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$ (c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$

SECTION-B

***This section consists of TWENTY* questions of *FIVE* marks each. *ANY FIFTEEN* out of these questions have to answered on the Answer Book provided.** **[75 Marks]**

3. Given $\vec{A} = y^2\hat{e}_x + 2yx\hat{e}_y + (xye^z - \sin x)\hat{e}_z$, calculate the value of $\iint_S (\nabla \times \vec{A}) \cdot \hat{n}\, ds$ over the part of the sphere $x^2 + y^2 + z^2 = 1$ above the *xoy* plane.

4. Find the matrix of the linear transformation T on $V_3(R)$ (i.e., three dimensional real vector space) defined as $T\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} a+b \\ b+c \\ c+a \end{pmatrix}$, with respect to the basis $B = \{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$, where $\hat{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\hat{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\hat{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

   Also calculate the matrix representation of T^{-1}.

5. Find the general solution of $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$, using the Frobenius power series method.

6. Consider the Lagrangian $L = \frac{q\ln(\dot{q})^2}{2} - \lambda q + qf(q)$, where $f(q)$ is an arbitrary function and $\lambda > 0$.
   - (a) Write down the Euler-Lagrange equation of motion
   - (b) Does the Lagrangian transform covariantly under the transformation $q \to \alpha q$ for α a real constant?
   - (c) Calculate the Hamiltonian of the system
   - (d) Is total energy a constant of motion?

7. A square lamina OABC of side *l* and negligible thickness is lying in the XOY plane of a Cartesian coordinate system such that O is at the origin and the sides OA and OC are along the positive X and Y directions respectively. Calculate the moment of inertia tensor and the directions of the three principal moments. The mass of the lamina is *m*.

8. A particle of mass *m* is subjected to a potential $V(x) = k|x|, k > 0$
   - (a) If H is the Hamiltonian of the particle, calculate $[H, V(x)]$
   - (b) Use the uncertainty principle in the form $\Delta x\, \Delta p \sim \frac{\hbar}{2}$ to estimate the ground state energy of the particle.

9. A particle of mass *m* in the one-dimensional energy well

   $$V(x) = \begin{cases} 0 & 0 \le x \le L \\ \infty & \text{elsewhere,} \end{cases}$$

   is in a state whose coordinate wave function is given by $\phi(x) = Cx(L - x)$, where *c* is the normalization constant.
   - (a) Determine the expectation value of the energy in the state $\phi(x)$
   - (b) Calculate the probability that on measurement of energy, the particle will be found in its ground state.

   $$\left[\text{Standard Integrals}: \int_0^L dx\, x\sin(\pi x/L) = L^2/\pi, \int_0^L dx\, x^2 \sin(\pi x/L) = \frac{L^3}{\pi} - \frac{4L^3}{\pi^3}\right]$$

10. Consider the harmonic oscillator in the form $H = (p^2 + x^2)/2$ (we have set $m = 1$, $\omega = 1$ and $\hbar = 1$). The harmonic oscillator is in its *n*th energy eigenstate and subjected to a time-independent perturbation $\lambda(xp + px)$, for λ real. Calculate the first-order energy shift and first-order correction to the wave function.

11. An ideal electron gas is confined to an area A in a two-dimensional plane at temperature T. Calculate
(a) the density of states
(b) N, the number of electrons
(c) E_F, the Fermi energy as a function of N

12. Write down the partition function of a particle of mass *m* whose potential energy is given by $V(x, y, z) = ax^2 + b(y^2 + z^2)^{1/2}$, where *a* and *b* are positive constants of suitable dimensions. Also calculate the average energy of the particle.

$$\left[\text{Standard Integral}: \int_{-\infty}^{\infty} dx\, e^{-x^2/2\beta^2} = \sqrt{2\pi}\,\beta\right]$$

13. Given that the molecular weight of KCl is 74.6 and its density is 1.99×10^3 kg/m^3, calculate the following:
(a) the distance between the atomic planes
(b) the lattice constant

14. The reaction $^3H(p,n)^2He$ has a Q value of –0.764 MeV. Calculate the threshold energy of incident protons for which neutrons are emitted in the forward direction.

15. A circular conducting loop C_1 of radius 2m is located in the XOY plane such that its centre is at (0, 0, 0). Another circular conducting loop C_2 of radius 2m is located at (0, 0, 4) such that the plane of C_2 is parallel to the XOY plane. A current of 5A is flowing in each of these loops such that the positive Z-axis lies to the left of the directions of the currents. Find the magnetic induction $\vec{B}$ produced at (0, 0, 0), neglecting the mutual induction of the loops.

16. Draw the electrical circuits for each of the following ************ source (battery), a detector (lamp), and switch (es).
(a) AND (b) OR (c) NOT (d) NAND (e) NOR

17. The pinch-down voltage of a *p*-channel junction FET is V_P = 5V and the drain-to-source saturation current I_{DSS} = –40mA. The value of drain-source voltage V_{DS} is such that the transistor is operating in the saturated region. The drain current is given as I_D = –15 mA. Find the gate-source voltage V_{GS}.

18. A narrow beam of electrons, accelerated under a potential difference, incident on a crystal whose grating space is 0.3 nm. If the first diffraction ring is produced at an angle 5.8º from the incident beam, find the momentum of the electrons and the potential difference applied.

19. The region $z > 0$ of a Cartesian coordinate system contains a linear isotropic dielectric of dielectric constant 2.0. The region $z < 0$ is the free space. A free space charge density of 5nC/m^2 is at the interface $z = 0$. If the displacement vector in the dielectric is $\vec{D}_2 = 3\hat{e}_x + 4\hat{e}_y + 6\hat{e}_z$ nC/m^2, find the corresponding displacement vector $\vec{D}_1$ in the free space.

20. The series limit of the Balmer series for hydrogen atom is given as 360 nm. Calculate the atomic number of the element that gives the lowest *x*-ray wavelength at 0.1 nm of the K-series.

21. The first few electronic energy states for neutral copper atom (Z = 29) are given as $E_1 < E_2 < E_3$, where E_1 being the ground electronic state. The states E_2 and E_3 are doubly degenerate due to spin splitting. Write the electron configuration of the states and arrange the spectral terms of the split levels following Hund's rules. Explain why $E_2 < E_3$.

22. The rotational lines of the CN band system at 3883.4Å is represented by a formula $\bar{\nu}$ = (25798 + 3.850m + 0.068 m^2) cm^{-1}, where *m* is a running number. Calculate the values of the rotational constants B'_v and B''_v, the location of the band head and the degradation of the band.